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NIM	A	B	C	D
SK006	4	9	2	7

1. $\int \frac{x}{(x+A)(x-B)^2}dx$

2. $\int \frac{x^3+Bx^2+D}{(x^2-C)^3}dx$

3. $\int \frac{1}{(x-C)\sqrt{(x^2-2Cx+1)}}dx$

4. $\int \frac{x^4-Bx^2+Cx-D}{Ax^2-C}dx$

5. $\int (A\sin^2 x - B\cos x + C)\sin x dx$

1. $\int \frac{x}{(x+4)(x-9)^2}dx$

2. $\int \frac{x^3+9x^2+7}{(x^2-2)^3}dx$

3. $\int \frac{1}{(x-2)\sqrt{(x^2-2.2x+1)}}dx = \int \frac{1}{(x-2)\sqrt{(x^2-4x+1)}}dx$

4. $\int \frac{x^4-9x^2+2x-7}{4x^2-2}dx$

5. $\int (4\sin^2 x - 9\cos x + 2)\sin x dx$

1. $\int \frac{x}{(x+4)(x-9)^2}dx$

$$\frac{x}{(x+4)(x-9)^2} = \frac{A}{(x+4)} + \frac{B}{(x-9)} + \frac{C}{(x-9)^2}$$

$x = \text{A } (x-9)^2 + \text{B } (x+4)\ (x-9) + \text{C } (x+4)$

untuk x = -4 → $-4 = \text{A } (-4-9)^2 + \text{B } (-4+4)\ (-4-9) + \text{C } (-4+4)$

$-4 = \text{A } (-13)^2 + \text{B } (0) + \text{C } (0)$

$-4 = \text{A } 169$

$169 \text{ A} = -4$

$\text{A} = -\frac{4}{169}$

untuk x = 9 → $9 = \text{A } (9-9)^2 + \text{B } (9+4)\ (9-9) + \text{C } (9+4)$

$9 = \text{A } (0) + \text{B } (0) + \text{C } (13)$

$9 = 13 \text{ C}$

$13 \text{ C} = 9$

$\text{C} = \frac{9}{13}$

untuk x = 8 → $8 = \text{A } (8-9)^2 + \text{B } (8+4)\ (8-9) + \text{C } (8+4)$

$8 = \text{A} + \text{B } (12)(1) + \text{C } (12)$

$8 = \text{A} + 12 \text{ B} + 12 \text{ C}$……………………………………………**(1)**

untuk x = 10 → $10 = \text{A} (10-9)^2 + \text{B } (10+4)\ (10-9) + \text{C } (10+4)$

$10 = \text{A} + \text{B } (14) + \text{C } (14)$

$10 = \text{A} + 14 \text{ B} + 14 \text{ C}$……………………………………………**(2)**

(2) $A + 14\,B + 14\,C \ = 10$

(1) $A + 12\,B + 12\,C \ = 8$

$$\underline{\hphantom{A + 14\,B + 14\,C \ = 10}} \quad -$$

$2\,B + 2\,C \ = \ 2$ ………… : 2

$B \ + \ C \ = \ 1$

$\rightarrow B \ + \ C \ = \ 1$

$B \ = \ 1 - C$

$$B \ = \ 1 - \frac{9}{13}$$

$$B \ = \ \frac{4}{13}$$

$$\frac{x}{(x+4)(x-9)^2} = -\frac{\frac{4}{169}}{(x+4)} + \frac{\frac{4}{13}}{(x-9)} + \frac{\frac{9}{13}}{(x-9)^2}$$

$$\int \frac{x}{(x+4)(x-9)^2}dx = \int -\frac{\frac{4}{169}}{(x+4)}dx + \int \frac{\frac{4}{13}}{(x-9)}dx + \int \frac{\frac{9}{13}}{(x-9)^2}dx$$

→ Misal $g(x) \ = x + 4$

$g'(x) \ = 1$

$$\int -\frac{\frac{4}{169}}{(x+4)}dx = -\frac{4}{169} \ \ln|x+4| + C$$

→ Misal $g(x) = x - 9$

$g'(x) = 1$

$$\int \frac{\frac{4}{13}}{(x-9)}dx = \frac{4}{13}\ \ln|\mathrm{x} - 9| + \mathrm{C}$$

$$\int \frac{\frac{9}{13}}{(x-9)^2}dx = \int \frac{\frac{9}{13}du}{u^2}$$

$$= \int \frac{9}{3}u^{-2}du$$

$$= \frac{9}{13}\frac{1}{-2+1}u^{-2+1} + C$$

$$= -\frac{9}{13}u^{-1} + C$$

$$= -\frac{9}{13}u^{-1} + C$$

$$= -\frac{9}{13u} + C$$

$$= -\frac{9}{13(x-9)} + C$$

$$\int \frac{x}{(x+4)(x-9)^2}dx = \int -\frac{\frac{4}{169}}{(x+4)}dx + \int \frac{\frac{4}{13}}{(x-9)}dx + \int \frac{\frac{9}{13}}{(x-9)^2}dx$$

$$\int \frac{x}{(x+4)(x-9)^2}dx = -\frac{4}{169}\ \ln|\mathrm{x}+4| + \frac{4}{13}\ \ln|\mathrm{x} - 9| - \frac{9}{13(x-9)} + C$$

2. $\int \frac{x^3 + 9x^2 + 7}{(x^2 - 2)^3} dx$

$$\int \frac{x^3 + 9x^2 + 7}{(x^2 - 2)^3} dx = \frac{Ax + B}{(x^2 - 2)} + \frac{Cx + D}{(x^2 - 2)^2} + \frac{Ex + F}{(x^2 - 2)^3}$$

$$x^3 + 9x^2 + 7 = (\mathrm{Ax} + \mathrm{B})(x^2 - 2)^2 + (\mathrm{Cx} + \mathrm{D})\ (x^2 - 2) + (\mathrm{Ex} + \mathrm{F})$$

$$\mathrm{x} = 0 \quad \rightarrow \qquad x^3 + 9x^2 + 7 \qquad = (\mathrm{A}0 + \mathrm{B})(0^2 - 2)^2 + (\mathrm{C}0 + \mathrm{D})\ (0^2 - 2) + (\mathrm{E}0 + \mathrm{F})$$

$$0^3 + 90^2 + 7 \qquad = (\mathrm{B})\ (-2)^2 + (\mathrm{D})\ (-2) + \mathrm{F}$$

$$7 \qquad = 4\mathrm{B} - 2\mathrm{D} + \mathrm{F}$$

$$4\mathrm{B} - 2\mathrm{D} + \mathrm{F} \qquad = 7$$ ……………………………………………………………**(1)**

$\mathrm{x} = \sqrt{2} \quad \rightarrow$

$$x^3 + 9x^2 + 7 \qquad = (\mathrm{A}\ \sqrt{2} + \mathrm{B})((\sqrt{2})^2 - 2)^2 + (\mathrm{C}\ \sqrt{2} + \mathrm{D})((\sqrt{2})^2 - 2) + (\mathrm{E}\ \sqrt{2} + \mathrm{F})$$

$$(\sqrt{2})(\sqrt{2})^2 + 9(\sqrt{2})^2 + 7 = (\mathrm{A}\ \sqrt{2} + \mathrm{B})(0) + (\mathrm{C}\ \sqrt{2} + \mathrm{D})\ (0) + (\mathrm{E}\ \sqrt{2} + \mathrm{F})$$

$$2(\sqrt{2}) + 9(2) + 7 \qquad = \mathrm{E}\ (\sqrt{2}\ ) + \mathrm{F}$$

$$2(\sqrt{2})\ + 18 + 7 \qquad = \sqrt{2}\ \mathrm{E} + \mathrm{F}$$

$$2(\sqrt{2})\ + 25 \qquad = \sqrt{2}\ \mathrm{E} + \mathrm{F}$$

$$\rightarrow \quad \mathrm{E}\cancel{\sqrt{2}} = 2\cancel{\sqrt{2}}$$

$$\mathrm{E} = 2$$ ……………………………………………………………**(2)**

$$\mathrm{F} = 25$$ ……………………………………………………………**(3)**

$x = 1 \rightarrow \quad (1)^3 + 9.(1)^2 + 7 = (A\,1 + B)((1)^2 - 2)^2 + (C\,1 + D)((1)^2 - 2) + (E\,1 + F)$

$1 + 9 + 7 = (A + B)(1) + (C + D)(-1) + (E + F)$

$17 = (A + B) - (C + D) + (2 + 25)$

$A + B - C - D = 17 - 27$

$A + B - C - D = -10$ ....**(4)**

$x = -1 \rightarrow$

$(-1)^3 + 9.(-1)^2 + 7 = (A(-1) + B)((-1)^2 - 2)^2 + (C(-1) + D)((-1)^2 - 2) + (E(-1) + F)$

$-1 + 9 + 7 = (-A + B)(1) + (-C + D)(-1) + (-E + F)$

$15 = (-A + B) + (C - D) + (-2 + 25)$

$-A + B + C - D = 15 - 23$

$-A + B + C - D = -8$ ....**(5)**

$x = 2 \rightarrow$

$(2)^3 + 9.(2)^2 + 7 = (A(2) + B)((2)^2 - 2)^2 + (C(2) + D)((2)^2 - 2) + (E(2) + F)$

$8 + 36 + 7 = (2A + B)\,(2)^2 + (2C + D)\,(2) + (2.2 + 25)$

$51 = 4\,(2A + B) + 2(2C + D) + (4 + 25)$

$51 = 8A + 4B + 4C + 2D + 29$

$8A + 4B + 4C + 2D = 51 - 29$

$8A + 4B + 4C + 2D = 22$

$4A + 2B + 2C + D = 11$ ....**(6)**

$$\begin{array}{lrl}
(1) \rightarrow & 4B - 2D + F & = 7 \\
 & 4B - 2D + 25 & = 7 \\
 & 4B - 2D & = 7 - 25 \\
 & 4B - 2D & = -18 \\
 & 2B - D & = -9
\end{array}$$

$$\begin{array}{lrll}
(4) \rightarrow & A + B - C - D & = -10 & \\
(5) \rightarrow & -A + B + C - D & = -8 & \\ \hline
 & & & + \\
 & 2B - 2D & = -18 & \\
 & B - D & = -9 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7) &
\end{array}$$

$$\begin{array}{lrll}
(1) \rightarrow & 2B - D & = -9 & \\
(7) \rightarrow & B - D & = -9 & \\ \hline
 & & & - \\
 & B & = 0 &
\end{array}$$

$$\begin{array}{lrl}
(7) \rightarrow & B - D & = -9 \\
 & 0 - D & = -9 \\
 & D & = 9
\end{array}$$

$$\begin{array}{lrll}
(4) \rightarrow & A + B - C - D & = -10 & \\
(5) \rightarrow & -A + B + C - D & = -8 & \\ \hline
 & & & - \\
 & 2A - 2C & = 2 & \\
 & A - C & = 1 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8) &
\end{array}$$

$$(6) \rightarrow \quad 4A + 2B + 2C + D = 11$$

$$4A + 2.0 + 2C + 9 = 11$$

$$4A + 2C = 11 - 9$$

$$4A + 2C = 2$$

$$2A + C = 1 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

$$(8) \rightarrow \quad A - C = 1$$

$$(9) \rightarrow \quad 2A + C = 1$$

$$\overline{\hphantom{2A + C = 1}} \; +$$

$$3A = 2$$

$$A = \frac{2}{3}$$

$$(8) \rightarrow \quad A - C = 1$$

$$\frac{2}{3} - C = 1$$

$$- C = 1 - \frac{2}{3}$$

$$C = -\frac{1}{3}$$

$$A = \frac{2}{3} \; ; B = 0 \; ; C = -\frac{1}{3} \; ; D = 9 \; ; E = 2 \; ; F = 25.$$

$$\int \frac{x^3 + 9x^2 + 7}{(x^2 - 2)^3} dx = \int \frac{\frac{2}{3} x}{(x^2 - 2)} dx + \int \frac{-\frac{1}{3} x + 9}{(x^2 - 2)^2} dx + \int \frac{2x + 25}{(x^2 - 2)^3} dx$$

$$\int \frac{\frac{2}{3}x}{(x^2 - 2)}dx$$

Misal : $u = x^2 - 2$

$du = 2x\ dx$

$$\int \frac{\frac{2}{3}xdx}{(x^2 - 2)} = \int \frac{\frac{1}{3}du}{u}$$

$$= \frac{1}{3}u + C$$

$$= \frac{1}{3}\ (x^2 - 2) + C$$

$$= \frac{1}{3}x^2 - \frac{21}{3} + C \text{ ....(1)}$$

Misal $u = x^2 - 2$

$du = 2x\ dx$

$dx = -\frac{1}{6}du$

$$\int \frac{-\frac{1}{3}x + 9}{(x^2 - 2)^2}dx = \int \frac{-\frac{1}{3}x}{(x^2 - 2)^2}dx + \int \frac{9}{(x^2 - 2)^2}dx$$

$$\int \frac{-\frac{1}{3}x}{(x^2-2)^2}\,dx = -\frac{1}{6}\int \frac{du}{u^2}$$

$$= (-\frac{1}{6})\frac{1}{-3+1}\;u^{-3+1} + \mathrm{C}$$

$$= (-\frac{1}{6})(-\frac{1}{2})\;u^{-2} + \mathrm{C}$$

$$= \frac{1}{12u^2} + \mathrm{C}$$

$$= \frac{1}{12(x^2-2)^2} + \mathrm{C}$$

Misal $\quad$ u = $x^2 - 2$

du = 2x dx

dx $= \frac{du}{2x}$

$$\int \frac{9}{(x^2-2)^2}\,dx = 9\int \frac{u^2 du}{2x}$$

$$= 9\frac{\frac{1}{3}u^3}{2.\frac{1}{2}x^2} + C$$

$$= 9\frac{1}{3}\frac{u^3}{x^2} + C$$

$$= 9\frac{1}{3}\frac{(x^2-2)^3}{x^2} + C$$

$$\int \frac{-\frac{1}{3}x+9}{(x^2-2)^2}dx = \int \frac{-\frac{1}{3}x}{(x^2-2)^2}dx + \int \frac{9}{(x^2-2)^2}dx$$

$$= \frac{1}{12(x^2-2)^2} + 9\frac{1}{3}\frac{(x^2-2)^3}{x^2} + C \text{ ....(2)}$$

Misal $u = x^2 - 2$

$du = 2x\ dx$

$$\int \frac{2xdx}{(x^2-2)^3} = \int \frac{du}{u^3}$$

$$= \frac{1}{-3+1}\ u^{-3+1} + C$$

$$= \frac{1}{-2}\ u^{-2} + C$$

$$= -2\ \frac{1}{u^2} + C$$

$$= -2\ \frac{1}{(x^2-2)^2} + C$$

Misal $u = x^2 - 2$

$du = 2x\ dx$

$dx = \frac{du}{2x}$

$$\int \frac{25dx}{(x^2 - 2)^3} = 25\int \frac{u^3 du}{2x}$$

$$= 25\frac{\frac{1}{4}u^4}{2.\frac{1}{2}x^2} + C$$

$$= 25\frac{1}{4}\frac{u^4}{x^2} + C$$

$$= 25\frac{1}{4}\frac{(x^2 - 2)^4}{x^2} + C$$

$$\int \frac{2x + 25}{(x^2 - 2)^3}dx = \int \frac{2xdx}{(x^2 - 2)^3} + \int \frac{25dx}{(x^2 - 2)^3}$$

$$= -2\frac{1}{(x^2 - 2)^2} + 25\frac{1}{4}\frac{(x^2 - 2)^4}{x^2} + C \text{ ....}\mathbf{(3)}$$

$$\int \frac{x^3 + 9x^2 + 7}{(x^2 - 2)^3}dx = \int \frac{\frac{2}{3}x}{(x^2 - 2)}dx + \int \frac{-\frac{1}{3}x + 9}{(x^2 - 2)^2}dx + \int \frac{2x + 25}{(x^2 - 2)^3}dx$$

$$= \frac{1}{3}x^2 - \frac{21}{3} + \frac{1}{12(x^2 - 2)^2} + 9\frac{1}{3}\frac{(x^2 - 2)^3}{x^2} + -2\frac{1}{(x^2 - 2)^2} + 25\frac{1}{4}\frac{(x^2 - 2)^4}{x^2} + C$$

3. $\int \frac{1}{(x-2)\sqrt{(x^2-2.2x+1)}}dx = \int \frac{1}{(x-2)\sqrt{(x^2-4x+1)}}dx$

$\sqrt{x^2-4x+1} \quad \rightarrow \; a > 0$

$a = 1$

- Misal $\sqrt{x^2-4x+1} = x\sqrt{a} + y$

  $\sqrt{x^2-4x+1} = x + y$

- $\left(\sqrt{x^2-4x+1}\right)^2 = (x+y)^2$

$$x^2-4x+1 = x^2+2xy+y^2$$

$$\cancel{x^2}-4x+1 = \cancel{x^2}+2xy+y^2$$

$$-4x-2xy = y^2-1$$

$$-2x(2+y) = y^2-1$$

$$x = \frac{y^2-1}{-2(2+y)}$$

$$x = \frac{-y^2-1}{2(2+y)}$$

$$x = -\frac{1}{2}\left(\frac{y^2-1}{y+2}\right)$$

$u = y^2-1 \quad \rightarrow \quad u' = 2y$

$v = y+2 \quad \rightarrow \quad v' = 1$

$$\frac{dx}{dy} = \frac{u'v - v'u}{v^2}$$

$$= -\frac{1}{2}\left(\frac{2y(y+2) - 1(y^2 - 1)}{(y+2)^2}\right)$$

$$= -\frac{1}{2}\left(\frac{2y^2 + 4y - y^2 + 1)}{(y+2)^2}\right)$$

$$= -\frac{1}{2}\left(\frac{y^2 + 4y + 1)}{(y+2)^2}\right)$$

$$\mathrm{dx} = -\frac{1}{2}\left(\frac{y^2 + 4y + 1)}{(y+2)^2}\right)dy$$

- $\sqrt{x^2 - 4x + 1} = \mathrm{x} + \mathrm{y}$

$$= \frac{1}{2}\left(\frac{y^2 - 1)}{(y+2)}\right) + y$$

$$= \frac{-\frac{1}{2}y^2 + \frac{1}{2}}{(y+2)} + \frac{y^2 + 2y}{(y+2)}$$

$$= \frac{\frac{1}{2}y^2 + 2y + \frac{1}{2}}{(y+2)}$$

$$= \frac{1}{2}\left(\frac{y^2 + 4y + 1}{y+2}\right)$$

- $$x-2 = -\frac{1}{2}\left(\frac{y^2-1)}{(y+2)}\right)-2$$

$$= \frac{-\frac{1}{2}y^2+1}{(y+2)} - \frac{2y+4}{(y+2)}$$

$$= \frac{-\frac{1}{2}y^2-2y-3}{(y+2)}$$

$$= -\frac{1}{2}\left(\frac{y^2-4y-6}{(y+2)}\right)$$

❖ $$\int \frac{1}{(x-2)\sqrt{(x^2-4x+1)}}dx = \int \frac{-\cancel{\frac{1}{2}}\left(\frac{y^2\cancel{+4y+1)}}{\cancel{(y+2)^2}}\right)dy}{-\cancel{\frac{1}{2}}\left(\frac{y^2-4y-6}{\cancel{(y+2)}}\right)\left(\frac{1}{2}\frac{y^2\cancel{+4y+1}}{\cancel{y+2}}\right)}$$

$$= \mathbf{2}\int \frac{1}{y^2-4y-6}dy$$

2b = 4 → b = 2

C = 6

P $= \sqrt{C-b^2}$

$= \sqrt{6-(2)^2}$

$= \sqrt{6-4}$

$= \sqrt{2}$

$$2\int \frac{1}{y^2 - 4y - 6}dy \;=\; 2\frac{1}{\sqrt{2}} \text{ arc tan } \frac{y-b}{\sqrt{2}} + C$$

$$= \; 2\frac{1}{\sqrt{2}} \text{ arc tan } \frac{y-2}{\sqrt{2}} + C$$

$$= \; \sqrt{2} \text{ arc tan } \frac{y-2}{\sqrt{2}} + C$$

- Karena $y = \sqrt{x^2 - 4x + 1} - x$

Maka $\quad = \quad \sqrt{2}\text{ arc tan } \dfrac{\sqrt{x^2 - 4x + 1} - x + 2}{\sqrt{2}} + C$

4. $\int \frac{x^4 - 9x^2 + 2x - 7}{4x^2 - 2}dx$

$$4x^2 - 2 \; \overline{\big) x^4 - 9x^2 + 2x - 7} \quad \text{quotient: } \frac{1}{4}x^2 - \frac{10}{4}$$

$$x^4 - \frac{2}{4}x^2 \qquad -$$

$$-\frac{34}{4}x^2 - 2x - 7$$

$$-\frac{34}{4}x^2 + \frac{18}{4} \qquad -$$

$$-2x - \frac{46}{4} \longleftarrow \text{(sisa)}$$

$$\int \frac{x^4 - 9x^2 + 2x - 7}{4x^2 - 2}dx = \int (\frac{1}{4}x^2 - \frac{10}{4}) - \frac{2x - \frac{46}{4}}{4x^2 - 2}dx$$

$$= \int \frac{1}{4}x^2 - \frac{10}{4}dx + \int \frac{2x - \frac{46}{4}}{4x^2 - 2}dx$$

- $\int \frac{1}{4}x^2 - \frac{10}{4}dx = \int \frac{1}{4}x^2 - \frac{10}{4}dx$

$$= \frac{1}{4}\frac{1}{3}x^3 - \frac{10}{4}x + C$$

$$= \frac{1}{12}x^3 - \frac{10}{4}x + C$$

$$\int \frac{2x - \frac{46}{4}}{4x^2 - 2}dx = \int \frac{2xdx}{4x^2 - 2} + \int \frac{-\frac{46}{4}}{4x^2 - 2}dx$$

Misal $g(x) = 4x^2 - 2$

$g'(x) = 8x$

$\rightarrow 2x = \frac{1}{4}(8x)$

$= \frac{1}{4}(g'(x))$

$$\int \frac{2xdx}{4x^2 - 2} = \int \frac{\frac{1}{4}(g'(x))}{g(x)}$$

$$= \int \frac{\frac{1}{4}(8x)}{4x^2 - 2}$$

$$= \frac{1}{4} \ln | 4x^2 - 2 | + C$$

$b = 0 ; c = 2$

$p = \sqrt{c - b^2}$

$= \sqrt{2 - 0}$

$= \sqrt{2}$

$$-\frac{46}{4}\int \frac{1}{4x^2 - 2}dx = \frac{1}{p} \text{ arc tan } (\frac{x + b}{p}) + C$$

$$= -\frac{46}{4}.\frac{1}{\sqrt{2}} \text{ arc tan } (\frac{x}{\sqrt{2}}) + C$$

$$\int \frac{x^4 - 9x^2 + 2x - 7}{4x^2 - 2}dx = \int (\frac{1}{4}x^2 - \frac{10}{4}) + \frac{2x - \frac{46}{4}}{4x^2 - 2}dx$$

$$= \int (\frac{1}{4}x^2 - \frac{10}{4}) - \int \frac{2x}{4x^2 - 2}dx + \int \frac{-\frac{46}{4}}{4x^2 - 2}$$

$$= \frac{1}{12}x^3 - \frac{10}{4}x + \frac{1}{4}\ln|4x^2 - 2| - \frac{46}{4}\cdot\frac{1}{\sqrt{2}} \text{ arc tan}(\frac{x}{\sqrt{2}}) + C$$

5. $\int (4\sin^2 x - 9\cos x + 2)\sin x dx$

$$\int (4\sin^2 x - 9\cos x + 2)\sin x dx = \int 4\sin^3 x dx - \int 9\cos x \sin x dx + \int 2\sin x dx$$

$$= 4\int \sin^3 x dx - 9\int \cos x \sin x dx + 2\int \sin x dx$$

- $2\int \sin x dx = 2(-\cos x) + C$

  $= -2\cos x + C$

- $9\int \cos x \sin x dx$

→ Cos x sin x $= \frac{1}{2}\{\sin(1x+1x) + \sin(1x-1)x\}$

$= \frac{1}{2}\{\sin(2x) + \sin(0)\}$

$= \frac{1}{2}\sin(2x)$

Misal $u = 2x$

$du = 2\ dx$

$dx = \frac{1}{2}\ du$

$$9\int \cos x \sin x dx = 9\int \frac{1}{2} \sin 2x dx$$

$$= 9\frac{1}{2}\int \sin 2x dx$$

$$= 9\frac{1}{2}\int \sin u \frac{1}{2} du$$

$$= 9\frac{1}{2}.\frac{1}{2}(-\cos u) + C$$

$$= -9\frac{1}{4}\cos 2x + C$$

$$= -\frac{9}{4}\cos 2x + C$$

- $4\int \sin^3 x dx = 4\int \sin^2 x \sin x dx$

Misal $u = \cos x$

$du = -\sin x\ dx$

$$\rightarrow 4\int \sin^2 x \sin x dx = 4\int (1 - \cos^2 x).(-\sin x dx)$$

$$= 4\int (1 - u^2).(-du)$$

$$= -4\int (1 - u^2) du$$

$$= 4\int (u^2 - 1) du$$

$$= 4\ \frac{1}{3}\ u^3 - u\ + \mathrm{C}$$

$$= \frac{4}{3}\ (\cos x)^3 - (\cos x)\ + \mathrm{C}$$

$$\int (4\sin^2 x - 9\cos x + 2)\sin x dx = \int 4\sin^3 x dx\ - \int 9\cos x \sin x dx\ + \int 2\sin x dx$$

$$= \frac{4}{3}\ (\cos x)^3 - (\cos x)\ -\ (-\ \frac{9}{4}\cos 2\mathrm{x}) + (-\ 2\ \cos \mathrm{x}) + \mathrm{C}$$

$$= \frac{4}{3}\ (\cos x)^3 - (\cos x)\ + \frac{9}{4}\cos 2\mathrm{x}\ - 2\ \cos \mathrm{x} + \mathrm{C}$$

$$= \frac{4}{3}\ (\cos x)^3\ + \frac{9}{4}\cos 2\mathrm{x} - 3\ \cos \mathrm{x} + \mathrm{C}$$